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MANIPULATION POWER IN BARGAINING GAMES USING MACHIAVELLIANISM

Abstract. *This paper suggests a novel approach for representing manipulation games based on Nash's bargaining model conceptualized under the Machiavellianism psychological theory. The Machiavellianism structure is represented by the concepts: views, tactics, and immorality. Such concepts determine a Stackelberg game consisting of manipulating and manipulated players which employ manipulation strategies to achieve power situations with the disposition to not become attached to a conventional moral. We consider a game model involving manipulating and manipulated players engaged cooperatively in a Nash bargaining game restricted to a Stackelberg game. As a result, we propose an analytical formula for solving the manipulation game, which arises as the maximum of the quotient of two Nash products. The solution of the manipulation game is a strong Stackelberg equilibrium. We represent the Stackelberg game as a Nash game for relaxing the interpretation of the non-cooperative bargaining solution and the equilibrium selection problem. The weights of the players for the Nash solution are determined by their role in the Stackelberg game. In the dynamics of the game, the manipulated players always break ties optimally for the manipulating players. The dynamics and the rationality proposed for the manipulation game correspond to many real-world manipulation situations. We fit the computation of the problem into a class of homogeneous, ergodic, controllable, and finite Markov chains games. A numerical example validates the usefulness of the method.*

Keywords: *Manipulation. bargaining Stackelberg game.*

JEL Classification: C72, C78

1.Introduction

1.1.Brief review

Manipulation is a fundamental social conduct that was modeled by research in economics, mathematics, and psychology [1,2]. We focus on employing the concept of

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manipulation based on Machiavellianism for representing non-cooperative bargaining games.

The Machiavellianism is defined as a strategy of social conduct that involves manipulating others for personal gain, often against the other's self-interest [3]. This concept coincides with the central insight that gave rise to modern economics where the common good is well served by the free actions of self-interested agents in a market. The profit maximization of agents assigns essentially no role to generosity and social conscience because actions in many domains of application commonly conform to standards of manipulation. In this sense, the manipulation model presents an advantage for expanding the classical economic models as a more realistic behavioral assumption. In the classical economic theory, agents are assumed to be rationally law-abiding, but not fair. This non-fairness assumption can be explained by the Machiavellianism in terms of the immorality (considered by to be among the three key elements of Machiavellianism) which has deep roots in the history of economy.

The concept of Machiavellianism was first studied by as the ability to manipulate others as an important personality trait. They analyze whether the principles associated with three of Machiavelli's greatest works (The Prince, The Discourses and The Art of War) were practiced by individuals in today's society. The fundamental idea throughout Machiavelli's discourses is the degree to which people can be manipulated.

Christie and Geis [3] defined the Machiavellian personality type as someone who seeks to manipulate others to achieve his or her own ends. Machiavellianism structure is composed by three key elements: 1) the belief in manipulative tactics, 2) a cynical world view, and 3) a pragmatic morality (immorality). For individuals who manipulate, others are viewed entirely as objects or as means to personal ends (views) having an utilitarian, rather than a moral view of their interactions with others (immorality) and focused on applying manipulation strategies for accomplishing their goals related to power situations (tactics).

The interest in the subject of Machiavellianism was among social psychologists. Wilson et al. [4] presented and approach of the Machiavellianism as a strategy in game theory models involving analogies with the conspiracy of doves, the prisoners' dilemma [5] and recursive alternatives of the same game. Gunnthorsdottir [6] found in experimental games employing Machiavellianism that particular individual choices which do not correspond to assumptions of material self-interest captured by the standard Nash equilibrium prediction. Sakalaki et al. [7] studied the differences between cooperators and defectors employing an evolutionary game theory approach employing Machiavellianism. In a previous work, Clempner [8] presented a game theory approach for modeling manipulation behavior based on the Machiavellianism psychological theory. The manipulation model is represented by a Stackelberg game employing a reinforcement learning approach for the implementation of the immorality concept providing a computational method, in which its principle of error-driven adjustment of cost/reward predictions contributes to the players' acquisition of moral (immoral) behavior. Clempner [22] proposed a model where parties involved are constrained both by adverse selection and moral hazard (immorality) restricted to a class of ergodic Bayesian-Markov problems.

The works mentioned above only made an interpretation of the concept of Machiavellianism related to well-known games of game theory and in game-theoretic experiments focusing on explaining the rationality of the players. These works have not converged the framework of game theory and a small number of articles are inspired by traditional game theory. In addition, the effects of repeated interactions with the intention to exploit others have been poorly addressed in the literature.

1.2. Main results

In this paper, we develop a completely new solution for the manipulation game based on the non-cooperative bargaining problem. The main results are summarized as follows.

- The manipulation game is conceptualized under the Machiavellianism psychological theory as a Stackelberg game model involving manipulating (leaders) and manipulated (followers) players.
- We consider a game model involving manipulating and manipulated players engaged cooperatively in a Nash game restricted by a Stackelberg game.
- The cooperation is represented by the Nash bargaining solution.
- An analytical method is proposed for finding the manipulation equilibrium point. There is a manipulating strategy solution (which arises as the maximum of the quotient of two Nash products) which under a feasibility condition is a manipulation equilibrium point (see Theorem 1).
- We represent the Stackelberg game model as a Nash game, for relaxing the interpretation of the game and the equilibrium selection problem.
- The solution concept applied to the manipulation game focuses on computing the manipulation equilibrium.
- Under conditions of unequal relative power among players, the player with high power tends to behave exploitative, while the less powerful player tends to behave submissively.
- The weights of the players for the Nash solution are determined by their role in the Stackelberg game.

- The manipulated players break ties optimally for the manipulating players finding a new strong Stackelberg equilibrium point solution where manipulating maximize the gain and the manipulated minimize the lost. There is an equilibrium selection problem forcing the manipulating players to manipulate on which equilibrium to converge.
- The manipulation equilibrium point is a strategy with an outcome, which is optimally better for the manipulating players.
- The computation of the problem is fitted into a class of homogeneous, ergodic, controllable, and finite Markov-chain games.

1.3.Organization of the paper

The remainder of this paper is organized as follows: Section 2 motivates the paper, presenting the bargaining game and the properties of manipulation game. Section 3 suggests the non-cooperative bargaining problem solution based on manipulation including the Machiavellianism structure and the manipulation solution of the game. A numerical example is presented in Section 4. Section 5 concludes the work with some remarks.

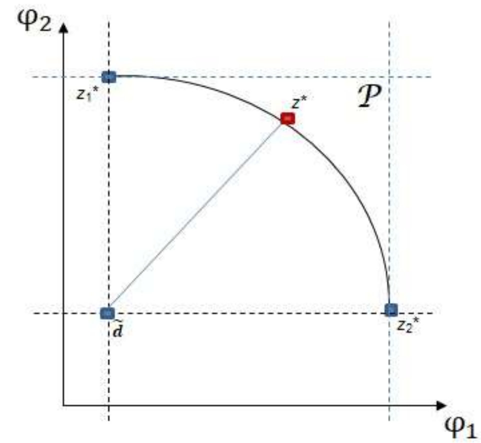

Figure 1. Bargaining

2.Motivation

Machiavellianism persists in game theory in such games where manipulation will be of interest to players in decision-making, like bargaining games. Nash established the framework to study bargaining where the players should cooperate when non-cooperation leads to Pareto-inefficient results. Bargaining game Γ (see Figure 1) is based on a model in which players are assumed to negotiate on a set of feasible pay-offs Φ. A fundamental element of the game is the disagreement point $\tilde{d}$ (status quo) which plays a role of a deterrent. A bargaining solution is a single-valued function that selects an outcome from the feasible pay-off for each bargaining problem which is the result of cooperation by the players involved in the game. The agreement reached in the game is the preferred alternative within the set Φ of feasible outcomes. Nash [9] proposed this approach by presenting four axioms and showing that they characterize the Nash bargaining solution [10, 11, 12, 13]. For a finite set of players $\mathcal{N}$ with *n* elements of a game Γ, a strategy z^* is called a *Nash bargaining solution* of Γ if z^* is an optimal solution of the maximization problem

$$\begin{aligned}
&\textstyle\prod_{l=1}^{n}(\varphi_l(z) - \tilde{d}_l) \to \max_z \\
&\text{subject to} \\
&(1)\ z \in \Phi, \\
&(2)\ \varphi_l(z) \geq \tilde{d}_l \text{ for all } l = 1,\ldots,n
\end{aligned}$$

where φ is the payoff and $\tilde{d}$ is the disagreement point. The pay-off $\varphi(z^*) = \left(\varphi_l(z^*)\right)_{l=\overline{1,n}}$ of players generated by the Nash bargaining solution z^* is *bargaining solution payoff*.

Game theory analyses of bargaining assume one of two approaches: a) the axiomatic, originates in the characterization of the Nash solution [9] (extended by [14]), where desired properties of a solution are satisfied, and b) the strategic, exemplified by Rubinstein's solution [15], where the bargaining procedure is modeled in detail as a sequential game. When players are patient, the equilibrium agreement of the Rubinstein's game approximates the agreement given by the Nash's axiomatic approach. In the bargaining problem the players have a mutual interest in reaching an agreement, although in general there is a conflict of interest over the particular agreement to be reached.

In the classical bargaining game theory models a bargainer has a positive interest in the others welfare as well as in his own. The agreement will represent a situation that could not be improved to both players' advantage. Rational players would not accept a given agreement if some alternative arrangement could make both parties better off or at least one better off with the other no worse off. Then, the resulting bargaining strategy is an outcome which is Pareto optimal. There are several papers related to the qualitative model of the Nash bargaining solution [16]. Literature related to rationality of bargaining solutions are the work of Clippel [17] which present sufficient conditions to ensure the rationality of the players in bargaining solutions.

The manipulation game is conceptualized under the Machiavellianism psychological theory which determines a Stackelberg game model consisting of manipulating and manipulated players that employ manipulation strategies to achieve power situations with the disposition to not become attached to a conventional moral [8,22]. The Stackelberg game focuses on computing the strong Stackelberg equilibrium [18, 19, 20, 10]. In this paper, we are considering manipulating and manipulated players engaged in a Nash's bargaining game. Power situations suggest that the advantage is for the manipulating players. The equilibrium may be imposed on the manipulated players without their approval but considering that every player is able of manipulative behavior to some degree (manipulated players try to minimize their lost). The resulting manipulation strategy is an outcome which is optimally better for the manipulating players with the manipulated players necessarily worse off. The rationality of the players follows these two basics principles: a) no manipulating player will agree to accept a payoff lower than the one guaranteed to him under disagreement, and b) the agreement will represent a situation that could not be improved by the manipulated players.

3.The manipulation game

3.1. Machiavellian structure

The *Machiavellianism structure* that encodes the set of characteristics of a Machiavellian individual is represented by three fundamental concepts [3]:

- Views: The belief that the world can be manipulated - the world consists of manipulating and manipulated players.
- Tactics: The use of a manipulation strategies needed to achieve specific power situations (goals).
- Immorality: The disposition to not become attached to a conventional moral.

Remark 1. *Strategies are based on the Machiavelli's The Prince, The Discourses, The Art of War, and the psychological behavior patterns* [3, 4].

A *manipulation game* is a Stackelberg game model consisting of manipulating and manipulated players (views) that employ manipulation strategies to achieve power situations (tactics) with the disposition to not become attached to a conventional moral (immorality) [8, 22].

The solution concept applied to the manipulation game is the strong Stackelberg Equilibrium. In the manipulation game, the manipulating players consider the best-reply of the manipulated players selecting the strategy that maximizes the payoff anticipating the predicted best-reply of the manipulated players. The manipulated players break ties optimally for the manipulating players and in equilibrium select the expected strategy as a best reply. We are considering manipulating and manipulated players engaged in a Nash's bargaining game restricted by a Stackelberg game.

The formal definition and rationality of the solution for the bargaining problem based on the manipulation game is as follows.

3.2.The bargaining manipulation solution

Let us consider a manipulation game, for a finite set of players $\mathcal{I} = \{\mathcal{N} \cup \mathcal{M}\}$ with $n+m$ elements, let $\mathbb{R}^{n+m}$ denote the $(n+m)$ -dimensional Euclidian space with coordinates indexed by the elements of $\mathcal{I}$. For representing a Stackelberg game as a Nash game, a strategy profile $z=(x,y) \in Z \subseteq R^{n+m}$ is constructed, where $Z = X \otimes Y$ is the concatenation of X and Y, such that $z = \left(x^1, \ldots, x^n, y^{n+1}, \ldots, y^{n+m}\right)$. The strategy profile x represents the proposal of the manipulating players and the strategy profile y represents the proposal of the manipulated players. The manipulation solution is based on a model in which the players are assumed to manipulate on which point of the feasible payoff vector $\overline{\Phi}(z) = \left(\varphi_1(x), \ldots, \varphi_n(x), \psi_{n+1}(y), \ldots, \psi_{n+m}(y)\right)$ where $\varphi(x) = \left(\varphi_1(x), \ldots, \varphi_n(x)\right)$ and $\psi\left(y\right) = \left(\psi_{n+1}(y), \ldots, \psi_{n+m}(y)\right)$ are the payoff vectors corresponding to the manipulating and manipulated players, respectively, and $z = \left(x^1, \ldots, x^n, y^{n+1}, \ldots, y^{n+m}\right) \in Z$. Let us denote $\Phi := \left\{\overline{\Phi}(z) \in \mathbb{R}^{n+m} \mid z \in Z\right\}$ as the adjunct set of payoff vectors $\overline{\Phi}(z)$.

Players have strictly opposed preferences and each one is concerned only with the share of benefits he/she obtains from manipulation. A fundamental point of the model is a fixed disagreement vector $\tilde{d} = (\tilde{\varphi}(x), \tilde{\psi}(y)) \in \Phi$ which plays the role of a deterrent where $\tilde{\varphi}(x) = (\tilde{\varphi}_1(x), \ldots, \tilde{\varphi}_n(x))$ is the disagreement payoff vector corresponding to the manipulating players and $\tilde{\psi}(y) = (\tilde{\psi}_{n+1}(y), \ldots, \tilde{\psi}_{n+m}(y))$ is the disagreement payoff vector corresponding to the manipulated players.

The manipulating players would like to increase their components in $\tilde{d}$ and to achieve a $\varrho \in \Phi(z)$ for which $\varrho \geq \tilde{d}$ (where $\varrho_i \geq \tilde{d}_i$, $i = 1,...,n+m$). We will suppose that the conflict of interest involves all external factors, depends only on the agreement being considered, and on the objections, and therefore the manipulation process is independent of time, history, and experience.

It is important to note that we are considering manipulating and manipulated players engaged in a (cooperative) Nash bargaining game restricted by a Stackelberg game (non-cooperative)

Remark 2. *In a Stackelberg game leaders and followers move asynchronously. For instance, if we first fix the followers then, we will have a Nash product for the leaders given by* $\prod_{l=1}^{n}(\varphi_l(x \mid y) - \tilde{\varphi}_l)$. *On the other hand, if we fix the leaders, we will have a Nash product for the followers given by* $\prod_{r=n+1}^{n+m}(\tilde{\psi}_r - \psi_r(y \mid x))$. *In a Stackelberg game we look for*

$$\max_{x \in X} \prod_{l=1}^{n}(\varphi_l(x \mid y) - \tilde{\varphi}_l),\ \varphi_l > \tilde{\varphi}_l$$

while

$$\min_{y \in Y} \prod_{r=n+1}^{n+m}(\tilde{\psi}_r - \psi_r(y \mid x)),\ \tilde{\psi}_r > \psi_r$$

Then, to fit the manipulation game to more real situations we consider that manipulating and manipulated players can move simultaneously. In addition, we consider that every player is capable of manipulative behavior to some degree but making emphasis in the fact that some are more willing and more able than others. We represent the Stackelberg game model as Nash game for relaxing the interpretation of the game and the equilibrium selection problem.

Following the Remark 2 we approach the solution of the bargaining by manipulation problem as the maximum of the quotient of two Nash products as follows.

Definition 3. *A strategy* $z^* = (x^*, y^*) \in Z$ *is called a manipulation strategy solution of the game* Γ *if it is an optimal solution of the maximization problem*

$$\max_{z \in Z} \quad \zeta(\varrho(z)) = \frac{\prod_{l=1}^{n}(\varphi_l(z) - \tilde{\varphi}_l)}{\prod_{r=n+1}^{n+m}(\tilde{\psi}_r - \psi_r(z))}$$

$$\text{subject to} \quad \varrho(z) \in \Phi(z) \qquad (1)$$

$$\varphi_l > \tilde{\varphi}_l \text{ and } \tilde{\psi}_r > \psi_r$$

where $\tilde{d}(z') = (\tilde{\varphi}(x), \tilde{\psi}(y)) \in \Phi(z')$, $z' \in Z$, $\varrho(z^*) = \left(\varphi\left(x^*\right), \psi\left(y^*\right)\right)$ *such that* $\varphi\left(x^*\right) = \left(\varphi_l\left(x^*\right)\right)_{l=\overline{1,n}}$ *, and* $\psi\left(y^*\right) = \left(\psi_r\left(y^*\right)\right)_{r=\overline{n+1,n+m}}$

Remark 4. *The pay-off vector given by* $\varrho(z^*) = \left(\varphi\left(x^*\right), \psi\left(y^*\right)\right)$ *generated by manipulation solution* $z^* = (x^*, y^*) \in Z$ *is called the manipulation solution payoff.*

The Eq. (1) for finding the solution to the manipulation problem can be rewritten as follows

$$\zeta(\varrho(z)) = \frac{\prod_{l=1}^{n}(\varphi_l - \tilde{\varphi}_l)^{\alpha^l \chi(\varphi_l > \tilde{\varphi}_l)}}{\prod_{r=n+1}^{n+m}(\tilde{\psi}_r - \psi_r)^{\beta^r \chi(\tilde{\psi}_r > \psi_r)}} \to \max_{z \in Z} \tag{2}$$

where $\varrho(z) = \left(\varphi\left(x\right), \psi\left(y\right)\right)$ and $\alpha^l \geq \beta^r > 0$ $(l = 1,..,n$, $r = n+1,..,n+m)$ are the weighting parameters for manipulating and manipulated players, respectively. Then, we rewrite the Eq. (2) as follows:

$$\begin{aligned} \zeta(\varrho(z)) = \sum_{l=1}^{n} \alpha^l \chi(\varphi_l > \tilde{\varphi}_l) \ln(\varphi_l - \tilde{\varphi}_l) - \\ \sum_{r=n+1}^{n+m} \beta^r \chi(\tilde{\psi}_r > \psi_r) \ln(\tilde{\psi}_r - \psi_r) \to \max_{z \in Z} \end{aligned} \tag{3}$$

where $\varrho(z) = \left(\varphi\left(x\right), \psi\left(y\right)\right)$ and $\varphi\left(x^*\right) = \varphi_l\left(x^*\right)_{l=\overline{1,n}}$ and $\psi\left(y^*\right) = \psi_r\left(y^*\right)_{r=\overline{n+1,n+m}}$.

Given $\zeta(\varrho(z))$ and considering the disagreement vector $\tilde{d}(z') = (\tilde{\varphi}(x), \tilde{\psi}(y)) \in \Phi(z')$, a payoff vector solution to the manipulation problem is a function $\varrho(z) \in \Phi(z)$ such that $z \in Z$. The manipulation process result in a particular strategy solution $z^* \in Z$ which can be considered the equilibrium point of the manipulation game when it results a particular point satisfying $\varrho(z^*) \in \Phi(z^*)$.

Definition 5. *The strategy solution* $z^* = (x^*, y^*) \in Z$ *of the manipulation game* Γ *is called the manipulation equilibrium point.*

In the following statement we present the characterization of the manipulation equilibrium point $z^* = (x^*, y^*) \in Z$ of the game Γ.

Theorem 6. *Let* Γ *be a manipulation game. Then, the manipulation strategy solution* $z^* = (x^*, y^*) \in Z$ *of* Γ *is a manipulation equilibrium point if and only if* $\varrho(z^*) \in \Phi(z^*)$.

Proof. $\Rightarrow$) Let us suppose that $z^* = (x^*, y^*) \in Z$ is an equilibrium point. In addition, let us suppose that there exists a $z \in Z$, $z \neq z^*$, where $\varrho(z) = \left(\varphi\left(x\right), \psi\left(y\right)\right) \in \Phi(z)$ such that $\varphi\left(x\right) > \varphi\left(x^*\right)$ and $\psi\left(y\right) > \psi\left(y^*\right)$. This is impossible, because $z^* \in Z$ is a solution of the manipulation game Γ. $\Leftarrow$) By contradiction. Let us suppose that $\varrho(z^*) \notin \Phi(z^*)$. Then, it is possible for the manipulating player to increase their pay-off. Consistently, it is possible for the manipulated players to reduce their pay-off. Then, it is not a manipulation equilibrium point (contradiction).

Remark 7. *The bargaining conditions under manipulation will produce that manipulating players prefers to increase the profit while the manipulated players prefer to decrease it. Under these circumstances, it may be necessary for all players to adjust the profit in order to find a strong equilibrium point solution. The change of the profit is also a strong equilibrium point solution because the renegotiated profit for the manipulated players is below the efficient profit.*

4.Numerical example

4.1. Markov games

Let $M = (S, A, \{A(s)\}_{s\in S}, P)$ be a Markov chain, where S is a finite set of *states*, $S \subset \mathbb{N}$ and A is the *set of actions*, which is a metric space. For each $s \in S$, $A(s) \subset A$ is the non-empty set of admissible actions at state $s \in S$. Without loss of generality we may take $A = \cup_{s\in S} A(s)$. Whereas, $\mathbb{K} = \left\{(s,a) \mid s \in S, a \in A(s)\right\}$ is the set of *admissible state-action pairs*, which is a measurable subset of $S \times A$. The variable $P = \left[p_{j|ik}\right]$ is a stationary controlled *transition matrix* which defines a stochastic kernel on S given $\mathbb{K}$, where $p_{j|ik} \equiv P(X_{t+1} = s_j \mid X_t = s_i, A_t = a_k)$ represents the probability associated with the transition from state s_i to state s_j,

$i = \overline{1,N}$ $(i = 1,...,N)$ and $j = \overline{1,N}$ ($j = 1,...,N$), under an action $a_k \in A(s_i)$, $k = \overline{1,K}$ $(k = 1,...,K)$. A *Markov Decision Process* is a pair $MDP = (M,C)$ where M is a controllable Markov chain and $C : \mathbb{K} \rightarrow \mathbb{R}$ is a *cost function*, associating to each state a real value.

A Markov game consists of a set $\mathcal{N} = \{1,...,n\}$ of players (indexed by $l = \overline{1,n}$). The dynamics of a Markov game is described as follows. Each of the players l is allowed to randomize, with distribution $\pi^l_{k|i}(t)$, over the pure action choices $a^l_k \in A^l\left(s^l_i\right)$, $i = \overline{1,N}$ and $k = \overline{1,K}$. From now on, we will consider only stationary strategies $\pi^l_{k|i}(t) = \pi^l_{k|i}$. In the ergodic case when all Markov chains are ergodic for any stationary strategy $\pi^l_{k|i}$ the distributions $P^l\left(X^l_{t+1} = s^l_j\right)$ exponentially (quickly) converge to their limits $P^l(s^l_j) = \sum\limits_{i=1}^{N}\left(\sum\limits_{k=1}^{K} p^l_{j|ik}\pi^l_{k|i}\right)P^l\left(s_i\right)$.

4.1. Manipulation example

Let us consider a two-player manipulation problem restricted to a class of ergodic controllable finite Markov chains where player 1 is the manipulating player and player 2 is the manipulated one. Let the states $N = 3$, and the number of actions $K = 2$.

We formulate the game in terms of nonlinear programming equations implementing the regularized Lagrange principle to ensure the convergence to an equilibrium point [21]. Applying the extraproximal method for finding the disagreement point (the Nash equilibrium point) we obtain the convergence of the strategies for the manipulating player and for the manipulated player (see Figure 2 and Figure 3).

The disagreement strategies are as follows:

$$x^1 = \begin{bmatrix} 0.0926 & 0.9074 \\ 0.4441 & 0.5559 \\ 0.7620 & 0.2380 \end{bmatrix} \qquad y^1 = \begin{bmatrix} 0.8250 & 0.1750 \\ 0.1637 & 0.8363 \\ 0.4694 & 0.5306 \end{bmatrix}$$

and the utilities at the disagreement point for each player are as follows:

$$\tilde{\varphi}_1 = 170.4446 \qquad \tilde{\psi}_1 = 97.1499$$

Now, the manipulation process begins. Fixing the weighting parameters $\alpha^1 = 70$ for the manipulating player and $\beta^1 = 30$ for the manipulated player and, applying the extraproximal method [21] we obtain the convergence of the strategies for the manipulating player and the convergence of the strategies for the manipulated player (see Figure 4 and Figure 5).
The resulting manipulation strategies are as follows:

$$x^1 = \begin{bmatrix} 0.5193 & 0.4807 \\ 0.5063 & 0.4937 \\ 0.4931 & 0.5069 \end{bmatrix} \qquad y^1 = \begin{bmatrix} 0.0384 & 0.9616 \\ 0.9939 & 0.0061 \\ 0.5545 & 0.4455 \end{bmatrix}$$

and utilities of the manipulation solution are as follows:

$$\psi^1(c^1) = 29.0885 \qquad \varphi^1(c^2) = 14.8154$$

As a result, the profit obtained for the manipulating player are greater than the disagreement point while for the manipulated player are smaller (see Figure 5).

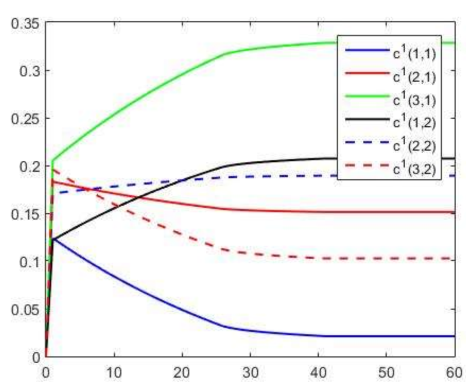

Figure 2: Strategies convergence manipulating player: disagreement.

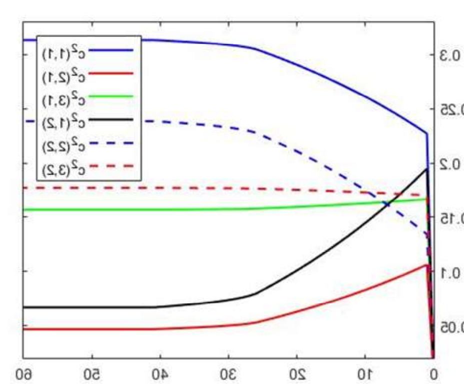

Figure 3: Strategies convergence manipulated player: disagreement.

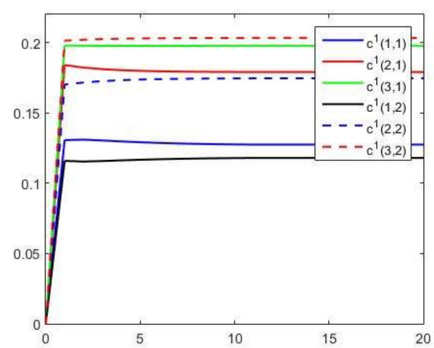

Figure 4: Strategies convergence manipulating player: manipulation.

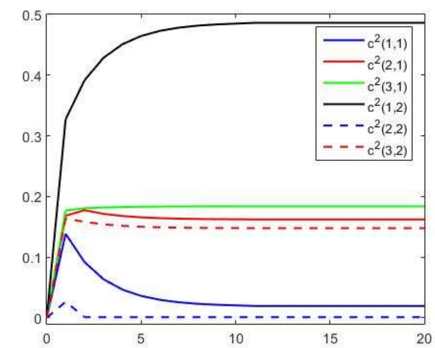

Figure 5: Strategies convergence manipulated player: manipulation.

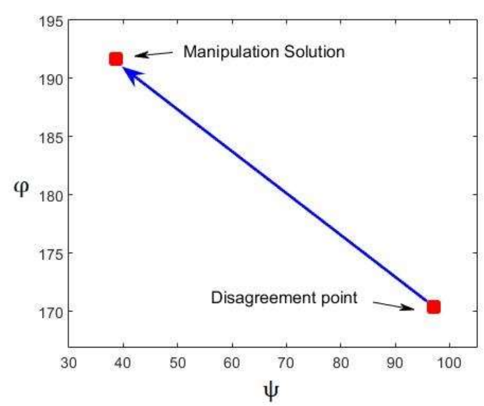

Figure 6: Manipulation Solution.

6.Conclusion and future work

This paper proposed a novel approach for representing manipulation games based on the Nash bargaining problem conceptualized under the Machiavellianism psychological theory. The Machiavellianism structure is represented by the concepts: views, tactics, and immorality. Such structure concepts determined a Stackelberg game consisting of manipulating (leaders) and manipulated (followers) players who employ manipulation strategies to achieve power situations with the disposition to not become attached to a conventional moral. We considered a game model involving manipulating and manipulated players engaged cooperatively in a Nash bargaining game, restricted by a Stackelberg game. The analysis of Stackelberg game focused on computing the strong Stackelberg equilibrium. We represented the Stackelberg game as a Nash game for relaxing the interpretation of the bargaining solution and the equilibrium selection problem. The weights of the players for the Nash solution were determined by the role of the players in the Stackelberg game. Then, conflict of interest arose about the agreement, and the manipulating players employed manipulation strategies to achieve new power situation. In the dynamics of the game, the manipulating players always break ties optimally for the manipulating ones finding a new strong Stackelberg equilibrium point solution where leaders and followers maximize the gain. The manipulation equilibrium point was established as a strategy with an outcome, which is optimally better for the manipulating players. We proposed an analytical formula for solving the manipulation game, which arose as the maximum of the quotient of two Nash products. The dynamics and the rationality proposed for the manipulation game correspond with many real-world manipulation situations. We fitted the computation of the problem into a class of homogeneous, ergodic, controllable, and finite Markov-chain games. A numerical example validated the proposed manipulation approach.

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